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The study highlights one class of systems of nonlinear difference equations with the continuous argument. We study the issue of existence of periodic solutions for such equations. We also obtain sufficient conditions for the existence of continuous N – periodic solutions (N is a positive integer) and propose the method for constructing such solutions. By employing the method of invariant manifolds, we prove the existence of continuous solutions for a system of nonlinear difference equations in the hyperbolic case as well as describe their structure in the vicinity of periodic solution.

In this article, we prove the formulas of inversion of integral presentation of p-analytical functions with characteristic $p = e^{\alpha x}$ та $p = e^{\alpha x} y^k$ (α, k – const>0). P-analytical functions with these characteristics are closely related to the Helmholtz equation and generalized axisymmetric Helmholtz equation fundamentally used for obtaining the integral presentation of $e^{\alpha x} y^k$-analytic functions by arbitrary analytic functions and their inversion formulas. Specifically, we determine the conditions, under which the direct and inverse formulas of integral presentation of $e^{\alpha x}$-analytical and $e^{\alpha x} y^k$-analytical functions are the solutions of integral equations of Volterra type.

In this paper, we tackle the group approach problem with unfixed time. The information on the initial function and the prehistory of the evader’s control is used in the course of the game. We propose the way of solving the problem with unfixed time where the evader’s mistakes can be used for reducing the time of approach. The game is considered to be over when an integral of some numeral function describing the course of the game is getting equal to the unit. Our research method is based on Minkowski’s inverse functionals of multi-valued maps closely related to the conflict-controlled process as well as on constructing the resolving functions. Additionally, L.S. Pontryagin’s condition is at the heart of the method’s scheme. It allows choosing pursuers’ control in the form of Borel’s measurable selections of the specific multi-valued map. Moreover, there is the period of switching from the method of resolving functions to the Pontryagin’s first direct method. Finally, we single out specific classes for differential-difference systems for which there is no such dependence.

The paper considers the problem of choosing phase operators corresponding to cosine and sine of the angle of harmonic waves in the classical approximation. According to the principle of correspondence of classical evolution equations for variable phase sine and cosine to their quantum analogues, we introduce phase operators. This definition is quite ambiguous, because there is a whole class of operators classically equivalent to phase variables, but essentially different in a quantum area. We show that a set of operators can be reduced to the unique pair of operators under general physical requirements. We also demonstrate that operators are unambiguously defined by the density of phase variable distribution in the vacuum state. We also specify the method for constructing the phase operator by using moments of the density distribution of the phase variable in the vacuum state. This method provides a convenient tool for calculating some phase characteristics. We show that the infinite sequence of the moments of density uniquely defines the phase operator. Additionally, we obtain matrices of phase operators in the basis of occupation numbers. We also find explicit expressions for calculating arbitrary moments of the operators and analyze their behavior.

Virchenko N.O.
To the Theory of the Generalized Integral Transform
P. 28–31.
Refs.: 7 titles.

Using the generalized confluent hypergeometric function, we describe a new generalization of integral transforms (Laplace, Stieltjes, the potential theory). Specifically, we study main properties of these new integral transforms (linearity, similarity). We find representations of generalized integral Laplace transforms of the unit and power functions. Some composition relations are proved. Relying on the tables of the classical integral transforms, they permit finding the representations of more composite functions. Parseval-type equalities are proved. These equalities allow calculating new integrals, which haven’t been yet described in the scientific literature.

Denysenko N.L.
The Study of the Structure of a Set of Continuously Differentiable on R^+ Solutions for Systems of Differential-Functional Equations with the Linearly Transformed Argument
P. 32–35.
Refs.: 7 titles.

In this paper, we consider the structure of a set of continuously differentiable on R^+ solutions for systems of linear inhomogeneous differential-functional equations with the linearly transformed argument. Crucially, we focus on equations with the delay. We utilize basic methods of the theory of ordinary differential and differential-functional equations as well as the method of successive approximations. Furthermore, we obtain new sufficient conditions of the existence of continuously differentiable on R^+ solutions for systems of linear differential-functional equations with the linearly transformed argument and develop the method for constructing such solutions.

Denysiuk V.P., Rybachuk L.V.
Polynomial Wavelets with Compact Support
P.36–40.
Fig. 3. Tabl. 1. Refs.: 5 titles.

We construct polynomial wavelets with compact support based on differentiating the proposed one-parametric basic finite polynomial wavelet of class C^m $(m = 0,1,\ldots)$. We show that derivatives of this basic wavelet of the order $k, 1 \le k < 2(m+1)$ in many cases are also wavelets, and the derivatives of the order $2(m+1)$ are nonreduced Haar functions. Some of the constructed

wavelets are orthogonal bases, and in case of nonorthogonality they are Riesz bases.

Dudkin M.E.

The Inverse Spectral Problem for the Block Jacobi-Type Matrices in the Complex Moments Problem in the Exponential Form

P. 41–44.

Refs.: 5 titles.

The article proposes the analog of Jacobi matrices related to the complex moments problem in the case of exponential form as well as to the system of orthonormal polynomials relative to some measure with the compact support on the complex plane. We obtain two matrices having block tridiagonal structure and acting in the space of l_2 type as a self-adjoint and unitary commuting operators. The previous research is incomplete as far as this research is concerned. Current are the research issues on the direct spectral problem, namely solving the system of difference equations generated by the obtained matrices. Of particular research interest is the investigation of the inner structure of such matrices, finding conditions for coefficients under which appropriate matrices are commutative self-adjoint and unitary operator.

Ilienko A.B., Fuior K.V.

The Limit Theorem for a Number of Collisions of Three Random Walks

P. 45–48.

Refs.: 11 titles.

In this paper, we investigate the limit theorem for a number of collisions of three independent simple non-symmetric random walks on the line. We obtain the limit theorem stating that the limit distribution of the log-normal number of collisions is the exponential one. Geometrically speaking, this theorem describes the asymptotic behavior of the cover time of the 3-dimensional simple random walk on the main diagonal $d := \{(x, y, z) \in \mathbb{Z}^3 : x = y = z\}$. This interpretation correlates our research results with the classical limit theorem for local and cover times of random walks. The research results can be used for statistical estimation of probability characteristics of the walks by the number of collisions observed. Specifically, it can be applied for building confidence intervals and testing statistical hypotheses for the parameter p.

Kapustyan V.E., Lazarenko I.S.

Optimal Control for Singular Perturbed Periodic Parabolic Equations with Nonlocal Boundary Value Conditions

P. 49–55.

Refs.: 7 titles.

This paper considers the issues of optimal control for singular perturbed by the spatial value linear parabolic equations with nonlocal boundary value conditions and the quadratic performance criterion. We construct the complete asymptotes of optimal solutions for the initial problem under optimal conditions by boundary functions method. Unlike similar problems for parabolic equations with local boundary value conditions, the iterative problems for boundary functions do not “decompose”. We prove that their solutions belong to the class of boundary functions, and complete decompositions are asymptotes of solutions of the corresponding order for the initial problem.

Kachanovsky N.O.

Clark–Ocone Type Formulas in the Meixner White Noise Analysis for Non-Differentiable by Hida Chance Quantities

P. 56–60.

Refs.: 16 titles.

Clark–Ocone type formulas allow presenting square integrable and differentiable by Hida chance quantities as stochastic integrals of some random processes as well as reconstructing a chance quantity by its Hida derivative. These formulas can be used in the stochastic analysis and in the financial mathematics. This paper aims to significantly extend a class of chance quantities, for which the Clark–Ocone type formulas in the Meixner white noise analysis can be applied. To this end, by employing the methods of the infinite-dimensional analysis as well as the theory of generalized functions, we show that the condition of differentiability by Hida in the classical sense when building the Clark–Ocone type formulas is actually not necessary at all. Hence applying Clark–Ocone type formulas is possible for square integrable by the generalized Meixner measure, but not differentiable by Hida chance quantities.

Lisetska O.M.
Integral Representations of Generalized Harmonic Functions
P.61–65.
Refs.: 7 titles.

We construct integral representations for *r*-generalized Legendre function and establish the connection with *r*-generalized Gauss function with integer values of parameters m and n. To this end, we utilize *r*-generalized Legendre functions of the first and second order, their connection with *r*-generalized hypergeometric function as well as some of its properties and integral representations. Finally, we prove two lemmas containing the connection formulas between *r*-generalized Legendre functions of the first and second order and *r*-generalized Gauss function with integer values of parameters m and n and integral representation for *r*-generalized Legendre function of the second order. The obtained results permit extending the field of application of *r*-generalized Legendre functions, in particular for solving boundary-value and other problems of mathematical physics.

Makasyeyev O.M.
Stability Regions of Fixed Points of Nonideal System "Pendulum-Electric Motor"
P. 66–70.
Fig. 5. Refs.: 6 titles.

In this paper, we investigate the nonideal dynamical system pendulum-electric motor. We construct and analyze the stability regions of equilibrium points. The study proposes a method of using such regions for searching the areas of deterministic chaos in the space of system parameters. Additionally, we demonstrate the falseness of applying the ideal mathematical models for studying the equilibrium points. Only the use of nonideal mathematical models, notably those models in which the oscillation source power is comparable to the power consumed by the oscillating system, allows us to make the right conclusion on the stability of equilibrium points.

Ovcharenko O.V.
Differential formulas for q-integral representation of (τ,β)-generalized hypergeometric function
P. 71–74.
Refs.: 11 titles.

The key objective of this paper is to obtain differential formulas for *q*-integral representation of (τ,β)-generalized hypergeometric function. To this end, we consider new (τ,β)-generalized hypergeometric functions ${}_2^qF_1^{\tau,\beta}(a,b;c;z)$ and ${}_3^qF_2^{\tau,\beta}(a,b_1,b_2;c_1,c_2;z)$. Using the integral property of q-beta function for the function ${}_3^qF_2^{\tau,\beta}(z)$, we obtain the q-integral representation. Employing the body of the theory of fractional differentiation, we also investigate a series of q-differential relations for these new generalized hypergeometric functions. The results obtained allow widely applying ${}_2^qF_1^{\tau,\beta}(z)$,

${}_3^qF_2^{\tau,\beta}(z)$ functions to solve problems of mathematical physics, differential and integral equations, the theory of probability, mathematical statistics and others.

Redko I.V., Snigur N.M.
Primitive Program Algebra of the Calculated Functions for Graphs
P. 75–80.
Refs.: 15 titles.

The problem of finding algebraic characteristics of representative classes of functions and predicates is closely connected with issues of the programming theory and practice. In this paper, we study a class of calculated functions and predicates for finite graphs. We choose graph structures because of their importance and popularity in the theoretical and applied programming. We also use the primitive program algebra as a research tool. Specifically, its carrier is a set of calculated functions and predicates for finite graphs and its signature comprises the parametric composition of superposition, branching and cycling. Emphasized here is finding a generating set of the primitive program algebra. In addition, we obtain useful necessary conditions for generating completeness of the generating set of the primitive program algebra calculated by functions and predicates for graphs.

Sivak O.A.
The Structure of a Set of Continuous Solutions of Systems of Linear Functional Difference Equations
P. 81–87.
Refs.: 4 titles.

This paper considers the structure of a set of systems of continuous solutions in a number of cases depending on the hypotheses for the matrices *A*, *B*, number *q* and their properties. Using the methods of the theory of differential and difference equations, we define new conditions for the existence of continuous solutions of these systems of equations. Specifically, we develop the method of their construction and examine their properties. In theorems 1 and 3 we obtain the results under the conditions $a_i > 1,\ i = 1, \ldots, n,\quad q > 1, (t \le 0),\quad 0 < a_i < 1,\quad i = 1, \ldots, m,\quad q > 1, (t \ge 0),$ whilst in theorems 5, 6 we obtain the research results under $0 < a_i < 1 < a_j,\ i = \overline{1, m},\ j = \overline{m+1, n},$ $0 \le m \le n,\ q > 1.$

Skuratovsky R.V.
Problem of Finiteness Conjecture and Joint Spectral Radius
P. 88–92.
Refs.: 4 titles.

This paper studies the constraint of a vector norm under periodic and aperiodic action of matrices from a finite set of matrices with rational elements as well as the presence of AZR and PZR. We consider a complex case when every matrix from M has its own numbers that are both bigger and smaller units. By solving the problem whether M has characteristics of AZR and PZR, we investigate a lower spectral radius (LSR) for such set M. The conducted research proves that PAS and AAS, are absent for the finite set of matrices $\tilde{M}$, which complies with specific conditions. We also determine that $\tilde{\rho}(\tilde{M}) \ge 1.$ We prove that AZR and PZR conditions for $\tilde{M}$ [□] and vectors from □ ^{n} are fulfilled. Finally, we determine that the systems of matrices $\tilde{M}$ are present above □, for which ES occurs.

Spichak S.V., Stogniy V.I., Kopas I.M.
Symmetry Properties and Exact Solutions of the Linear Kolmogorov Equation

P. 93–97.
Refs.: 14 titles.

This paper considers the symmetry properties of the linear Kolmogorov equation. We obtain the maximal invariance algebra of this equation. Moreover, we classify all two-dimensional subalgebras of the invariance algebra up to action of transformations of its automorphism group. Using the obtained subalgebras, we reduce the symmetry to ordinary differential equations and separate variables for this equation. In some cases we integrate the reduced equations and to obtain exact solutions of the linear Kolmogorov equation.

Shkliar T.B.
Global Attractor of Nonautonomous Evolution Inclusion of Reaction-Diffusion Type
P. 98–104.
Refs.: 9 titles.

The present paper considers the nonautonomous evolution inclusion of reaction-diffucion type, whose right part is majorized by continuous functions of step growth, for which additional conditions of translation compactness are imposed. We prove the existence and study the properties of the global attractor of the family of multivalued processes generated by solutions of the inclusion. Relying on the solutions of nonautonomous inclusion with the right part of power growth, we construct the family of multivalued processes. In addition, we prove the existence of the invariant stable connected global attractor for this family in the phase space. It comprises bounded completed trajectories. This proof method can be applied to other classes of problems: evolutional inclusions of the second order and systems of phase-field equations with the multivalued function of interaction.

Yuzhakova G.O.
Composition Formulas for the τ-Generalized Gauss Hypergeometric Function
P. 105–110.
Refs.: 5 titles.

The paper deals with the τ-generalized (by Wright) Gauss hypergeometric function ${}_2F_1^{\tau}(a, b; c; z)$. We formulate and prove the lemma of composition relations for ${}_2F_1^{\tau}(a, b; c; z)$ and its adjacent functions. Our proof is based on using the representation of ${}_2F_1^{\tau}(a, b; c; z)$ function in the series form as well as employing some properties of the classic gamma function. The proved formulas generalize known expressions for classic Gauss hypergeometric function ${}_2F_1(a, b; c; z)$, a specific case of the considered τ-generalized function ${}_2F_1^{\tau}(a, b; c; z)$ when $\tau = 1$.

Yaremenko N.I.
Solvability of Quasi-Linear Elliptic Equation with Gilbar–Serrin Matrix in the Space Scale R^l
P. 111–117.
Refs.: 15 titles.

The study under scrutiny proves that it's possible to solve the nonlinear differential equation in the second-order partial derivatives with operator coefficients in all Euclidean space R^l, in the spaces scale ${W_1}^p$. We also propose a new class of operators associated with the given differential equation. We construct the nonlinear semigroup of compression in L^2 for specific differential operators $A : D(A) \to L^2(R^l, d^l x)$, generated by the left part of these differential equations. Furthermore, we describe some possible topological constructions in $L_{L_2(R^l, d^l x)}([0, t])$, instrumental in proving the

analogue of the Hille–Yosida–Phillips theorem. Specifically, we demonstrate that operators introduced in the elliptic equation are actually local generators of semigroups.

Anisimova O.V.

On Determining the Parameters of Stationary Centrosymmetrical Low-Pressure Glow Discharge

P. 118–123.

Fig. 4. Refs.: 8 titles.

The paper defines the spatial distribution of parameters of the stationary centrosymmetrical low-pressure glow discharge. To solve this problem, we use the system of nonlinear differential equations comprising charged-particle flow equations taking account drift and diffuse components, Poisson equations on intensity of the electrical field. We propose and implement in software the iteration algorithm for solving the self-consistent system of differential equations that insure reducing the problem to solving the first-order Cauchy problem, which allow minimizing the influence of "network diffusion". Furthermore, we determine spatial distribution of the intensity of electrical field and the density of charged particles. We study the influence of pressure and of the value of electron temperature as well as of the length of the discharge gap and diffusion processes on the glow discharge characteristics and the dependence of spatial distributions from diffuse processes. The obtained results are consitent with main principles of the classical theory of glow discharge.

Bushinsky V.O., Voronov S.O., Pankratov V.I., Rodionov V.M.

Correlation Analysis of Characteristics Gloss and Smoothness of Paper Tape with Purpose of Their Control in Technological Process

P. 124–128.

Fig. 3. Tabl. 1. Refs.: 6 titles.

Through experiments conducted, we study the characteristics of gloss and smoothness of paper tape to develop the method of their technological control. By utilizing the method of statistical analysis, we prove that there is nonlinear correlation dependence between them. We calculate the parameters of regression equation, which has a parabolic form. Moreover, we find that for certain angles of incidence of light ray 70–80°, the parameter of correlation communication is the correlation relation signifying 0,80–0,85. It allows proposing the optical method of technological control of the parameter smoothness of paper directly in the process of production instead of the existent laboratory pneumatic method of measuring (the Bekks method).

Gerasimchuk I.V.

Localization of the Light Beam in the System of Two Nonlinear Optical Waveguides

P. 129–132.

Refs.: 8 titles.

Theoretically, we investigate the character of localization of nonlinear stationary waves propagating along an array of two identical nonlinear optical waveguides in a linear medium. The Kerr nonlinearity is taken into account in the waveguides, and the medium between the waveguides is considered to be optically linear. We investigate the solutions of the corresponding equation for the bypass nonlinear monochromatic wave at the presence of two delta-function perturbations. We also study the stationary localized states of light beams propagating in systems of two plane-parallel nonlinear optical waveguides. In addition, we show that the problem can be reduced to the model of coupled anharmonic oscillators. Moreover, all characteristics of such system are found. We calculate a total number of excitations in the wave and the total energy of the system for all three types of possible stationary states: the in-phase symmetric state with the

same intensity of light fluxes in both waveguides, the antisymmetric state with the same flux density in the waveguides, but with the opposite wave phases in them, and the inhomogeneous state with the same phases but different densities of light flux in the waveguides.

Kulish V.V.
Single-Electron Optical Properties of Nanoeggs
P. 133–138.
Refs.: 21 titles.

The aim of this paper is to investigate single-electron optical properties of a spherical nanoegg comprising a dielectric core and a thin metal shell with a slight shift of the core center relative to the geometric center of the nanoparticle. Furthermore, we propose a model for this type of the composite nanoparticle. It allows calculating the wave functions and the electron wavenumber spectrum of the electron in the metal shell of the nanoegg. Specifically, the contribution of the latter dominates in the optical properties of the whole particle. Using this model, we obtain the matrix elements of optical transitions and an optical conductivity of the nanoegg in a quasi-classical approximation. The conductivity obtained differs from the conductivity of a spherical nanoshell by an amount squarely proportional to the magnitude of the core center shift relative to the geometric center of the nanoparticle as well as inversely proportional to the square of the average thickness of the shell. The obtained expressions fall within the domain of the frequency range where the contribution of a single-electron component is essential.

Lukianov P.V.
Model of the Quasi-Point Vortex
P. 139–142.
Fig. 3. Refs.: 8 titles.

In this paper, we obtain the nonviscous model of compact vortex close to the classical point vortex. The obtained vortex differs from the classical point vortex flow; its azimuthal velocity field is not potential. In addition to the classical flow solution, there is a term that makes the vortex compensated. The overall vorticity in it equals zero. Unlike the classical point vortex, the obtained one is compact and its velocity field meets the non-slip condition at the outer boundary like the (Taylor-Couette) flow between two coaxial cylinders. Furthermore, we introduce the non-dimensional parameter, the function of radial coordinate indicating locally how the quasi-point vortex differs from the point vortex. The proposed model is an alternative to the point vortex one. The quasi-point vortex can be used in all relevant problems where its counterpart (the point vortex) was used before. Crucially, its flow domain will always have finite scales just like it’s in the nature.

Lukianov Petro V.
Sound Generation by Helicopter Blade Swept by Subsonic Flux
P. 143–148.
Fig. 4. Refs.: 11 titles.

The article studies the sound characteristics (rotational noise) of the helicopter blade swept by the subsonic flux. To this end, we analyze the existing theoretical models of the rotational noise and determine their differences as well as the limits of their use. We choose the theory of the small disturbances spreading off the thin wing was chosen as a calculation model. Using this model, we solve the problem of the sound generation by the blade of the helicopter rotor. We also study the generated noise level for different thicknesses of the rotor blade. Specifically, we determine that the shape of the generated sound wave depends on both contrary flux velocity and the cross section thickness of the blade. The analysis of the obtained calculation data and its conformity with experimental results indicate that this model is quite sufficient (under the stated hypotheses) for rotational noise simulation of the rotor blade.

Ol’shanskii S.V.

The Calculation of Droplet’s Vertical Motion that Evaporates under the Sreznevsky Law

P. 149–153.

Fig. 2. Tabl. 1. Refs.: 10 titles.

By employing the Airy functions, we find the solution of the nonlinear Cauchy problem describing the vertical motion of the droplet evaporating under the Sreznevsky law. Using a special variable transformation in the differential equation of motion, we find the first integral with the cylindrical functions. This transformation has increased the order of the differential equation, but it became linear with variable coefficients. We also propose an approximate solution for calculating the displacement. Moreover, we compare theoretical and experimental results: they are consistent with the physical view on the process of vertical particles motion of variable mass and comply with research results of other authors. After defining the constants of the formula model, it’s possible to calculate kinematic characteristics of the droplet evaporating under specific conditions of the flight.

Reshetnyak S.A., Berezhinskiy A.S.

Parameters of Surface Bifocal Spin-Wave Lens

P. 154–158.

Fig. 5. Refs.: 12 titles.

The paper investigates the process of spin waves refraction when passing through the inhomogeneous structure, a biaxial ferromagnet in the form of the biconvex lens placed in the medium of an uniaxial ferromagnet. We theoretically calculate the dependencies of “optical” parameters (refractive index, focal length) on such spin-wave lens. The paper uses geometrical optics formalism to describe the behavior of surface spin waves propagating in the ferromagnetic medium with nonuniform distribution of magnetic parameters. The obtained frequency and field dependencies of surface spin-wave lens focal lengthes for different branches of spin waves demonstrate the ability to manage their relative values by changing frequency and magnitude of external homogeneous magnetic field.